



## Technical Communication

## Effect of load inclination on the undrained bearing capacity of surface spread footings above voids



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## ABSTRACT

Footings are often situated on ground that includes voids that were either undetected or had not formed at the time of construction. In this study, small-strain finite element analyses are conducted to investigate the influence of load inclination on the bearing capacity of surface spread footings on undrained homogeneous clay with single and double continuous voids. The numerical solutions are compared with existing theoretical and empirical predictions. The results are presented as failure envelopes expressed in terms of loads that are non-dimensionalised by the footing width and the undrained soil shear strength (these are known as bearing capacity factors) and loads that are normalized by the uniaxial ultimate load. If underground voids exist under the footing, the shape and the size of the failure envelope defining the undrained bearing capacity of the footing subjected to combined horizontal and vertical loading depends on the location, the geometry and the number of voids.

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## 1. Introduction

Underground voids occur as a result of the dissolution of soluble material in response to hydrochemical changes in the soil and/or rock [1]. A karstic cavity in a limestone formation/deposit is a typical example of mineral dissolution. Unstable voids may also form because of human activity such as mining or leakage from a faulty sewer [2]. The presence of underground voids beneath a rigid foundation system has a significant effect on the stability of the foundation, which can lead to extensive damage and loss of life.

Several studies have been undertaken to evaluate the stability of voids embedded within cohesive-frictional soil using model tests [3–6], finite element simulations [7–11], or plasticity limit analyses [12,13]. These analyses concentrated on the vertical footing capacity because the vertical load on a footing, which is mainly due to the weight of the superstructure, is the most important. However, for footings under inclined loading, the previous studies may not provide reliable predictions.

The objective of this study is to explore the effect of load inclination on the bearing capacity of spread footings on the surface of homogeneous, purely cohesive soil with voids. The effects of the location, the shape and the number (i.e., one or two) of continuous voids are studied. Finite element solutions are obtained, and the

results are presented as normalized failure envelopes in the horizontal and vertical loading planes.

## 2. Background

For undrained conditions, the bearing capacity of a rigid spread footing at the surface subjected to inclined loading can be approximated as

$$q_u = c_u N_c \zeta_i \quad (1)$$

where  $q_u$  is the ultimate (average) bearing stress on the footing,  $c_u$  is the undrained shear strength of the soil,  $N_c$  is the bearing capacity factor for cohesion, and  $\zeta_i$  is the load inclination factor. For pure vertical loading, the load inclination factor has a value of 1 and the solution is equal to the Prandtl solution [14].

Table 1 summarizes the most commonly used expressions for the load inclination factor, which were derived by Hansen [15], Meyerhof [16] and Vesic [17] using limit analysis, a theoretically rigorous method for calculating the bearing capacity of footings. Green [18] suggested a plane-strain plasticity solution for a surface footing under inclined loading given by

$$\frac{V}{Bc_u} = \left(1 + \frac{\pi}{2}\right) + \cos^{-1} \left(\frac{H}{Bc_u}\right) + \sqrt{1 - \left(\frac{H}{Bc_u}\right)^2} \quad (2)$$

where  $B$  is the width of the footing and  $V$  and  $H$  are the vertical and horizontal components of the applied load, respectively. For a

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